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A QCD sum rule study of the light scalar mesons

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Abstract. We examine the interpretation of the light scalar meson nonet as bound states of the scalar diquark and the scalar antidiquark using the QCD sum rule approach. Our results are obtained by means of the operator product expansion (OPE) including operators up to dimension 8. They show no evidence of the coupling of the tetraquark states to the light scalar meson nonet.

PACS. 11.55.Hx Sum rules – 12.38.Aw General properties of QCD (dynamics, confinement, etc.) – 12.38.Lg Other nonperturbative calculations

1 Introduction

The quasi-bound scalar diquark is one of the main candidates as a building block of stable multiquark systems [1]. Both, perturbative one-gluon exchange [2] and non-perturbative instanton dynamics [3] favor the existence of such clusters inside conventional and exotic hadrons. Maiani *et al.* [4] associate the unusual properties of the light scalar nonet of mesons $\sigma(600)$, $\kappa(800)$, $f_0(980)$ and $a_0(980)$ to their structure as bound states of diquark and antidiquark.

It is important to obtain a justification for such tetraquark picture from QCD. Recently, the QCD sum rule techniques [5] were used in papers by Brito *et al.* [6] and Wang *et al.* [7] to calculate the decays and masses of the members of the light scalar meson nonet in a tetraquark picture. Their calculation took only into account the contributions from operators up to dimension $d = 6$ in the OPE. In [8,9] it was shown that, for multiquark systems, potentially important contributions to the QCD sum rules may arise from the operators of higher dimension $d > 6$ and, if not considered, wrong conclusions about the properties of the exotic hadrons might be drawn.

In this paper we apply the QCD sum rules (SR) technique to the light scalar meson nonet described as systems composed of the scalar diquark and the scalar antidiquark as done in [6]. Using the factorization hypothesis for calculating the OPE, we show that the contribution from the operators of dimension 8 is dominant and leads to the destruction of SR. We find no evidence for the coupling of the above structure of diquarks to the light scalar meson nonet within the QCD SR approach.

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2 QCD sum rules for the scalar nonet

In the picture of the tetraquark states, the light scalar meson nonet is generated by the diquark in the $\bar{\mathbf{3}}_f$ and the antidiquark in the $\mathbf{3}_f$, where the subscript f stands for flavor. The diquark and the antidiquark are assumed to belong to $\bar{\mathbf{3}}_c$, $\mathbf{3}_c$ in color space and to spin-zero state. Their conventional wave functions in flavor space are given by

$$\begin{aligned}
&\sigma(600) = [ud][\bar{u}\bar{d}], \quad f_0(980) = \frac{1}{\sqrt{2}}\Big([us][\bar{u}\bar{s}] + [ds][\bar{d}\bar{s}]\Big),\\
&a_0^+(980) = [us][\bar{d}\bar{s}], \quad a_0^0 = \frac{1}{\sqrt{2}}\Big([us][\bar{u}\bar{s}] - [ds][\bar{d}\bar{s}]\Big),\\
&a_0^- = [ds][\bar{u}\bar{s}], \quad \kappa^+(800) = [ud][\bar{d}\bar{s}], \quad \kappa^0 = [ud][\bar{u}\bar{s}],\\
&\bar{\kappa}^0 = [us][\bar{u}\bar{d}], \quad \kappa^- = [ds][\bar{u}\bar{d}],
\end{aligned} \tag{1}$$

where the square bracket represents the normalized antisymmetric diquark (antidiquark) state [1].

From this structure, the interpolating current for the scalar nonet in eq. (1) can be written as

$$J_S = N_S \epsilon_{abc}\epsilon_{ade}(q_{1b}^T \Gamma q_{2c})(\bar{q}_{3d}\tilde{\Gamma}\bar{q}_{4e}^T)\,, \tag{2}$$

where $\tilde{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$ and N_S is the normalization constant. Here, the indices $a, b, c, \ldots$ denote color and the subscripts $1, 2, 3, 4$ are introduced for flavor. The index S labels each meson in the scalar nonet. ϵ_{abc} and ϵ_{ade} guarantee that

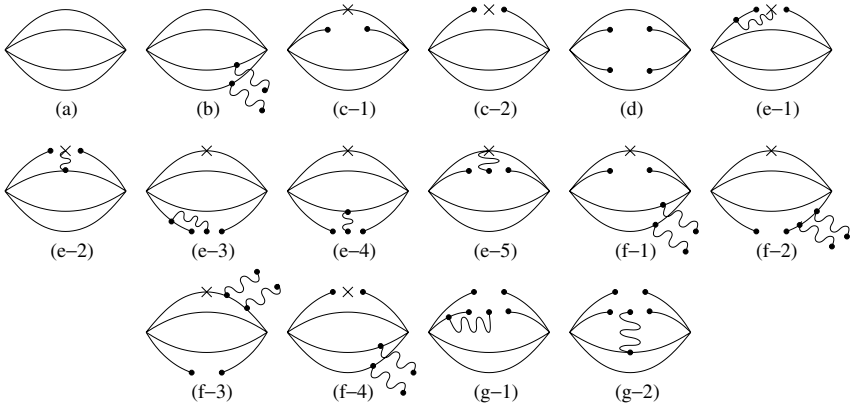

Fig. 1. Diagrams contributing to the sum rule in eq. (6) with the mass of the strange quark denoted by ×.

the diquark and the antidiquark belong to $\bar{\mathbf{3}}_c$ and $\mathbf{3}_c$, respectively. The antisymmetric structure of the nonet in both flavor and color spaces requires that the spin matrix Γ must have the following property:

$$\Gamma^T = -\Gamma \tag{3}$$

under the transpose of the spin indices. Here we take $\Gamma = C\gamma_5$ in order to consider the scalar diquark-antidiquark system. The interpolating currents for each meson in the nonet read

$$\begin{aligned}
J_\sigma &= \epsilon_{abc}\epsilon_{ade}(u_b^T C\gamma_5 d_c)(\bar{u}_d C\gamma_5 \bar{d}_e^T)\,,\\
J_{f_0} &= \frac{1}{\sqrt{2}}\epsilon_{abc}\epsilon_{ade}\Big((u_b^T C\gamma_5 s_c)(\bar{u}_d C\gamma_5 \bar{s}_e^T)\\
&\quad +(d_b^T C\gamma_5 s_c)(\bar{d}_d C\gamma_5 \bar{s}_e^T)\Big)\,,\\
J_{a_0^0} &= \frac{1}{\sqrt{2}}\epsilon_{abc}\epsilon_{ade}\Big((u_b^T C\gamma_5 s_c)(\bar{u}_d C\gamma_5 \bar{s}_e^T)\\
&\quad -(d_b^T C\gamma_5 s_c)(\bar{d}_d C\gamma_5 \bar{s}_e^T)\Big)\,,\\
J_{\kappa^+} &= \epsilon_{abc}\epsilon_{ade}(u_b^T C\gamma_5 d_c)(\bar{d}_d C\gamma_5 \bar{s}_e^T)\,,
\end{aligned} \tag{4}$$

where the overall negative sign from the identity $\tilde{\Gamma} = -\Gamma$ for $\Gamma = C\gamma_5$ is ignored.

We consider the correlator of the currents to get the QCD SR for each meson:

$$\Pi_S(q) = i\int \mathrm{d}^4x\; e^{iq\cdot x}\langle 0|TJ_S(x)J_S^\dagger(0)|0\rangle\,. \tag{5}$$

Within the narrow resonance approximation, including the operators up to dimension 8, after Borel transforming, we obtain the QCD SR for each meson which can be written in the form,

$$\begin{aligned}
&C_{0,1}^S M^{10}E_4\Big|_{(a)} + C_{4,1}^S g_c^2\langle G^2\rangle M^6 E_2\Big|_{(b)}\\
&+\Big(C_{4,2}^S m_s\langle\bar{q}q\rangle + C_{4,3}^S m_s\langle\bar{s}s\rangle\Big)M^6E_2\Big|_{(c)}\\
&+\Big(C_{6,1}^S\langle\bar{q}q\rangle^2 + C_{6,2}^S\langle\bar{q}q\rangle\langle\bar{s}s\rangle\Big)M^4E_1\Big|_{(d)}\\
&+\Big(C_{6,3}^S m_s ig_c\langle\bar{s}\sigma\cdot Gs\rangle E_1 + m_s ig_c\langle\bar{q}\sigma\cdot Gq\rangle\\
&\times(C_{6,4}^S E_1 + C_{6,5}^S\overline{W}_1)\Big)M^4\Big|_{(e)}\\
&+m_s g_c^2\langle G^2\rangle\Big(\langle\bar{q}q\rangle(C_{8,1}^S E_0 + C_{8,2}^S W_0) + C_{8,3}^S\langle\bar{s}s\rangle E_0\Big)M^2\Big|_{(f)}\\
&+\Big(C_{8,4}^S\langle\bar{q}q\rangle ig_c\langle\bar{s}\sigma\cdot Gs\rangle + C_{8,5}^S\langle\bar{s}s\rangle ig_c\langle\bar{q}\sigma\cdot Gq\rangle\\
&+C_{8,6}^S\langle\bar{q}q\rangle ig_c\langle\bar{q}\sigma\cdot Gq\rangle\Big)M^2E_0\Big|_{(g)} = 2f_S^2 m_S^8 e^{-m_S^2/M^2},
\end{aligned} \tag{6}$$

where M is the Borel mass. The decay constant and the mass of the mesons of the scalar nonet are defined by

$$\langle 0|J_S|S\rangle = \sqrt{2}f_S m_S^4\,. \tag{7}$$

In fig. 1 the diagrams which contribute to the sum rules are shown. The contribution from the continuum is encoded in the functions $E_n(M)$, $W_n(M)$, and $\overline{W}_n(M)$ [8] defined by

$$\begin{aligned}
E_n(M) &= \frac{1}{\Gamma(n+1)M^{2n+2}}\int_0^{s_0^2}\mathrm{d}s^2\; e^{-s^2/M^2}(s^2)^n\,,\\
W_n(M) &= \frac{1}{\Gamma(n+1)M^{2n+2}}\int_0^{s_0^2}\mathrm{d}s^2\; e^{-s^2/M^2}(s^2)^n\\
&\times\big(-2\ln(s^2/\Lambda^2)+\ln\pi+\psi(n+1)+\psi(n+2)+2\gamma_E-2/3\big)\,,\\
\overline{W}_n(M) &= \frac{1}{\Gamma(n+1)M^{2n+2}}\int_0^{s_0^2}\mathrm{d}s^2\; e^{-s^2/M^2}(s^2)^n\\
&\times\big(-2\ln(s^2/\Lambda^2)+\ln\pi+\psi(n+1)+\psi(n+2)+2\gamma_E\big)\,,
\end{aligned} \tag{8}$$

where s_0 is the threshold of the continuum and $\psi(n) = 1 + 1/2 + \cdots + 1/(n-1) - \gamma_{EM}$. The first index in the coefficients $C_{d,n}^S$ denotes the dimension in powers of energy of the operators. In order to get the contributions from the operators of dimension 6 and 8, we use the factorization

hypothesis which is based on $1/N_c$ arguments [10]. Here we would like to point out that we do not include $\langle g_c^3 GGG\rangle$ in the operators of dimension 6 and $\langle g_c^2 G^2\rangle^2$ in the operators of dimension 8 since they are produced in OPE of higher orders in the expansion in the strong coupling constant. Furthermore, it is well known [11,12] that contribution to QCD SR from pure gluonic operators is small for the hadrons consisting of light quarks. Note that thanks to the structure of the interpolating currents, the sum rules for $f_0(980)$ and $a_0(980)$ are the same. The coefficients in the sum rules for each meson are the following:

1. σ:

$$C_{0,1}^{\sigma} = \frac{1}{2^9\cdot 5\pi^6}\,, \qquad C_{4,1}^{\sigma} = \frac{1}{2^{10}\cdot 3\pi^6}\,,$$
$$C_{6,1}^{\sigma} = \frac{1}{12\pi^2}\,, \qquad C_{8,6}^{\sigma} = -\frac{1}{12\pi^2}\,, \tag{9}$$

and the others vanish.

2. f_0 and a_0:

$$C_{0,1}^{f_0,a_0} = \frac{1}{2^9\cdot 5\pi^6}\,, \qquad C_{4,1}^{f_0,a_0} = \frac{1}{2^{10}\cdot 3\pi^6}\,,$$
$$C_{4,2}^{f_0,a_0} = -\frac{1}{2^4\cdot 3\pi^4}\,, \qquad C_{4,3}^{f_0,a_0} = \frac{1}{2^5\cdot 3\pi^4}\,,$$
$$C_{6,1}^{f_0,a_0} = 0, \qquad C_{6,2}^{f_0,a_0} = \frac{1}{12\pi^2}\,,$$
$$C_{6,3}^{f_0,a_0} = \frac{1}{2^7\cdot 3\pi^4}\,, \qquad C_{6,4}^{f_0,a_0} = \frac{1}{2^6\pi^4} = C_{6,5}^{f_0,a_0}\,,$$
$$C_{8,1}^{f_0,a_0} = -\frac{5}{2^7\cdot 9\pi^4}\,, \qquad C_{8,2}^{f_0,a_0} = -\frac{1}{2^6\cdot 3\pi^4}\,,$$
$$C_{8,3}^{f_0,a_0} = \frac{1}{2^8\cdot 3\pi^4}\,, \qquad C_{8,4}^{f_0,a_0} = -\frac{1}{24\pi^2} = C_{8,5}^{f_0,a_0}\,,$$
$$C_{8,6}^{f_0,a_0} = 0\,. \tag{10}$$

3. κ:

$$C_{0,1}^{\kappa} = \frac{1}{2^9\cdot 5\pi^6}\,, \qquad C_{4,1}^{\kappa} = \frac{1}{2^{10}\cdot 3\pi^6}\,,$$
$$C_{4,2}^{\kappa} = -\frac{1}{2^5\cdot 3\pi^4}\,, \qquad C_{4,3}^{\kappa} = \frac{1}{2^6\cdot 3\pi^4}\,,$$
$$C_{6,1}^{\kappa} = \frac{1}{24\pi^2} = C_{6,2}^{\kappa}, \qquad C_{6,3}^{\kappa} = \frac{1}{2^8\cdot 3\pi^4}\,,$$
$$C_{6,4}^{\kappa} = \frac{1}{2^7\pi^4} = C_{6,5}^{\kappa}, \qquad C_{8,1}^{\kappa} = -\frac{5}{2^8\cdot 9\pi^4}\,,$$
$$C_{8,2}^{\kappa} = -\frac{1}{2^7\cdot 3\pi^4}\,, \qquad C_{8,3}^{\kappa} = \frac{1}{2^9\cdot 3\pi^4}\,,$$
$$C_{8,4}^{\kappa} = -\frac{1}{48\pi^2} = C_{8,5}^{\kappa}, \qquad C_{8,6}^{\kappa} = 2C_{8,4}^{\kappa}\,. \tag{11}$$

Before finishing this section let us comment on the possible deviation of the numerical values of the condensates of dimension 6 and 8 from their factorization values. This issue was discussed in recent papers [13,14] using the OPE expansion for the $V \pm A$ correlators and data from e^+e^- annihilation and hadronic τ decays. We point out that the color and the Dirac structure of our condensates of dimension 6 and 8 are different from the analysis of the OPE of the $V \pm A$ correlators so that it is difficult to use directly their results in our case. But even if we assume that the ratios of violations of the factorization hypothesis, in our case, for the condensates of dimension 6 and 8, are similar to those presented in [14], our final conclusion will not change due to the dominant contribution from the condensate of dimension 8 to the sum rules. Unfortunately, since there is no experimental data to be applicable to the present condensates, we do not have any other choice but the factorization hypothesis in calculating them.

3 Numerical results

For the numerical analysis, we use the following values of the parameters [6]:

$$m_s = 0.13\,\text{GeV}, \qquad \langle\bar{u}u\rangle = -(0.23)^3\,\text{GeV}^3,$$
$$f_s = \frac{\langle\bar{s}s\rangle}{\langle\bar{u}u\rangle} = 0.8, \qquad ig_c\langle\bar{q}\sigma\cdot Gq\rangle = 0.8\,\text{GeV}^2\langle\bar{q}q\rangle,$$
$$g_c^2\langle G^2\rangle = 0.5\,\text{GeV}^4. \tag{12}$$

Comparing the strength of the coefficients and the numerical values of the various condensates, one can see, in the left-hand side (LHS) of the sum rules eq. (6), that the operators of dimension 6 and 8 give the main contributions. More precisely, the contributions from the operators of dimension 6 and the last three operators of dimension 8 with the coefficients $C_{8,4}^S$, $C_{8,5}^S$, and $C_{8,6}^S$ dominate the sum rule. Furthermore, the contribution from the $d = 8$ operators comes with opposite sign to that from the dimension $d = 6$ operators in the physical region of Borel mass $M \approx 1\,\text{GeV}$, and the former becomes larger enough to overcome the latter.

In figs. 2, 3 and 4, the LHS of the sum rules eq. (6) up to the operators of dimension 4 (short-dashed line), 6 (long-dashed line), and 8 (solid line) as a function of the Borel mass M for $f_0(980)$, $a_0(980)$, $\sigma(600)$, and $\kappa(800)$ are shown with the thresholds, $s_0^{f_0} = 1.22\,\text{GeV}$, $s_0^{\sigma} = 1.0\,\text{GeV}$, and $s_0^{\kappa} = 1.1\,\text{GeV}$ [6,7], respectively.

As shown in the figures, the most dominant contribution comes from the operators of dimension 8: consequently, the QCD SR cannot have physical meaning because the LHS is negative definite. One could think that

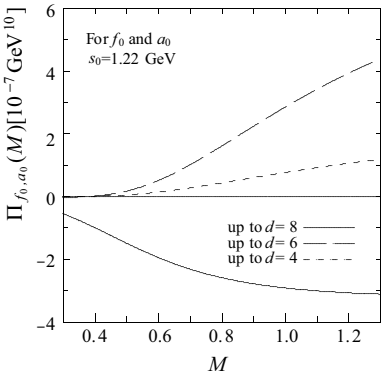

Fig. 2. The left-hand side of the QCD sum rule for $f_0(980)$ and $a_0(980)$ with the scalar diquark and the scalar antidiquark.

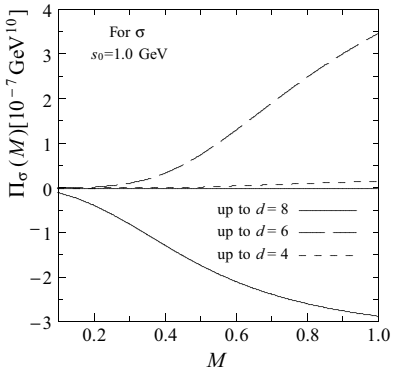

Fig. 3. The left-hand side of the QCD sum rule for $\sigma(600)$ with the scalar diquark and the scalar antidiquark.

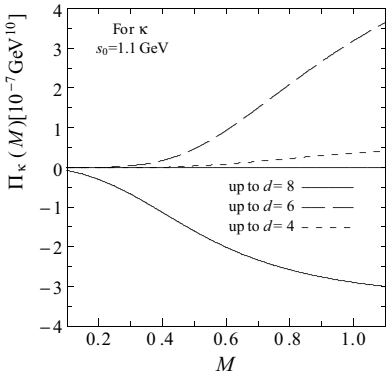

Fig. 4. The left-hand side of the QCD sum rule for $\kappa(800)$ with the scalar diquark and the scalar antidiquark.

contributions from higher-dimensional operators $d > 8$ might lead to stabilization of the QCD SR. However, we mention that the contribution from the operators of dimension 10 to the QCD SR is constant. They have the form of $g_c^2\langle G^2\rangle\langle\bar{q}q\rangle^2$, $(ig_c\langle\bar{q}\sigma\cdot Gq\rangle)^2$, and $m_s\langle\bar{q}q\rangle^3$ with the factorization hypothesis. Since their values are small, their contribution to the QCD SR are expected to be very small. The next operators are of dimension 12, 14, ... They appear with powers of M^{-2} in the sum rules and therefore their contribution is expected to be small in region where $M \approx 1\,\mathrm{GeV}$.

4 Conclusion

Our main conclusion is that we do not find a justification for the interpretation of the light mesons in the scalar nonet as the scalar diquark-antidiquark bound states within the QCD SR approach. We have demonstrated that the contribution of the operators of dimension 8 with the factorization hypothesis is very large and leads to the disappearance of the coupling of the tetraquark states to the light scalar meson nonet. Of course, this conclusion might change if another type of interpolating currents is considered or the factorization is broken in some unexpected way. The investigation of the properties of tetraquark states within the QCD SR approach with other interpolating diquark currents, *e.g.*, the pseudoscalar diquark, the vector diquark, or with some mixture of $q\bar{q}$ configurations, is in progress [15].

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